Quiz 6A, Calculus I - No Calculators

Dr. Graham-Squire, Spring 2014

Name: Key

1. (4 points) Let $h(x) = \int_4^x \left(\frac{1}{t} - e^t + \frac{1}{1+t^2}\right) dt$.

   (a) Use an antiderivative to find an expression for $h(x)$ that does <u>not</u> involve an integral. Leave your answer in exact form (no decimals).

   (b) Use the Fundamental Theorem of Calculus (or your answer from (a)) to find $h'(x)$.

(a) $h(x) = \int_4^x \left(\frac{1}{t} - e^t + \frac{1}{1+t^2}\right) dt$

$= \ln|t| - e^t + \arctan t \Big|_4^x$ ✓✓✓

$\boxed{h(x) = \ln|x| - e^x + \arctan x - (\ln 4 - e^4 + \arctan(4))}$ ✓✓

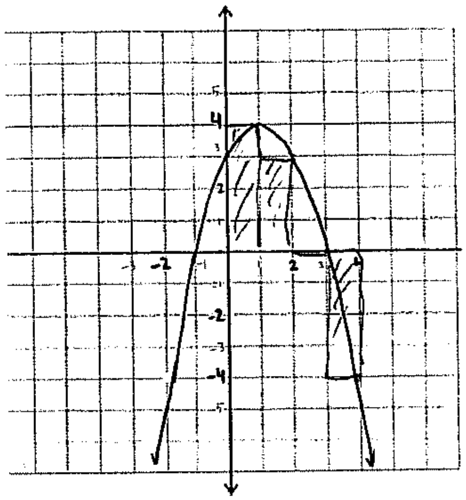
(b) $\boxed{h'(x) = \frac{1}{x} - e^x + \frac{1}{1+x^2}}$ by F.T.C. (or by taking $\frac{d}{dx}$ of

✓ ✓ ✓

2. (3 points) (a) Use right endpoints and 4 subintervals (that is, find the Riemann sum R_4) to approximate the area under the curve $f(x) = 4 - (x-1)^2$ on the interval [0,4].

(b) Is your answer an overestimate or an underestimate? Explain your reasoning.

$\frac{4}{4} = 1$



(a) $\text{Area} = 1\left(f(1) + f(2) + f(3) + f(4)\right)$

$= 1\left(4 + 3 + 0 + (-5)\right)$

$= \boxed{2}$

(b) underestimate because 1st rectangle is an overestimate but the others are all underestimates.

Check: $\int_0^4 4 - (x^2 - 2x + 1)\, dx$

$= \int_0^4 (-x^2 + 2x + 3)\, dx$

$= -\frac{x^3}{3} + x^2 + 3x \Big|_0^4 = -\frac{4^3}{3} + 16 + 12 - 0 = -\frac{64}{3} + 28 = \frac{20}{3}$ (bigger than 2)

3. (3 points) Calculate the indefinite integral (that is, the most general antiderivative) of

$$\int \frac{x + x^4 3^x + x^8}{x^4}\, dx.$$

$$= \int \left(\frac{x}{x^4} + \frac{x^4 3^x}{x^4} + \frac{x^8}{x^4}\right) dx$$

$$= \int \left(x^{-3} + 3^x + x^4\right) dx$$

$$= \boxed{\frac{x^{-2}}{-2} + \frac{3^x}{\ln 3} + \frac{x^5}{5} + C}$$